TABLE II. Adiabatic second-order elastic constants of columbium obtained in the present and in previous investigations. The investigators of Refs. 24 and 27 used ultrasonic methods and Refs. 25 and 28 used the resonance method. The values for the present samples listed without parentheses were determined directly from the measured ultrasonic wave velocities and the other values were calculated from them. $C_S' = (C_{11} - C_{12})/2$, $C_L' = (C_{11} + C_{12} + 2C_{44})/2$, $K = (C_{11} + 2C_{12})/3$, and $A = C_{44}/C_S'$.

Present	Temp. (°K)	density (g/cm³)	C_{11} [a]	C_{12} [a]	C_{44} [a]	C_S' [a]	C_L' [a]	K [a]	A
Present									
Sample 1	298	8.578	(2.4653)	(1.3335)	0.28368	0.56592	2.1831	(1.7108)	(0.5013)
Sample 2	298	8.578	2.4645	(1.3323)	0.28431	0.56618	2.1828	(1.7098)	(0.5022)
"Best" values	298	8.578	(2.465±0.005)	(1.333±0.007)	0.2840±0.0006	0.5661	2.1829	(1.7102)	(0.5017)
Previous									
Ref. 24	300	8.578	2.456±0.0098	1.345±0.014	0.2873±0.0011	0.5604	2.187	1.718	0.5127
Ref. 27	300	8.5605	2.456±0.015	1.387±0.46	0.2930±0.0018	0.5345	2.215	1.743	0.5482
Ref. 25	298	8.578	2.34	1.21	0.2821±0.0004	0.571	2.06	1.59	0.495
Ref. 28	RT	...	2.40±0.11	1.26±0.11	0.2809±0.0007	0.57	2.11	1.64	0.493

[a] Units of 10^{12} dyn/cm².

in the calculational equation

$$m_n = [F(C_{ij})/\Delta p](2\Delta f/f_0). \quad (2)$$

This equation was used to calculate the value of the slope m for each of the runs. Uncertainty limits for the slopes were established based on the estimated uncertainty in Δf and in the stress, p. Examples of a hydrostatic pressure and a uniaxial stress run are shown in Figs. 1 and 2.

Because of the redundancy in the number of relations available to determine the values of the single-crystal TOEC, and the wide range of uncertainties in the values of m_n, the data analysis from this point is highly subjective. Several procedures were tried with only slightly different results, so only one of these are described. The hydrostatic pressure data was considered the most reliable and was found to have the best internal consistency based on the relations $m_2 \equiv m_5$, and $m_1 + m_2 = m_3 + m_4$, which can readily be shown. The hydrostatic pressure equations were then solved

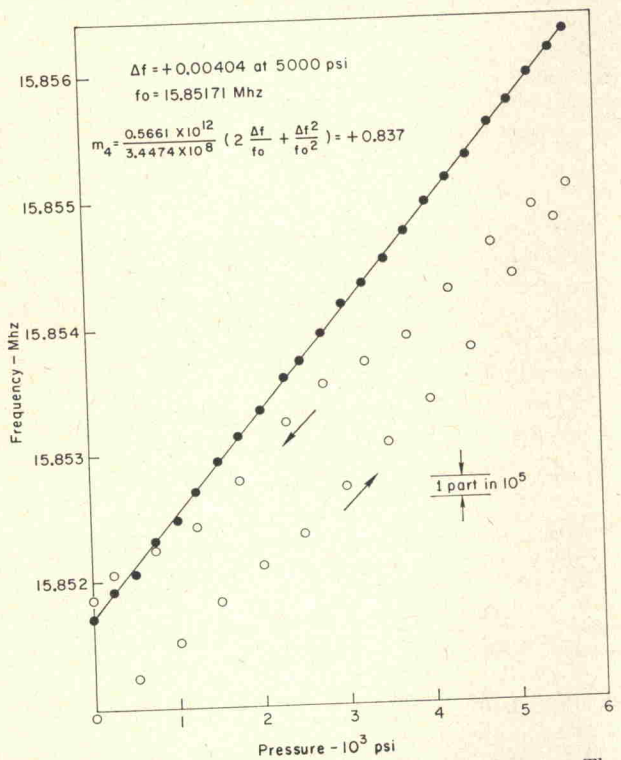

FIG. 1. Example of data for a hydrostatic pressure run. The open circles are data before correcting for temperature changes during the run. The temperatures at the start, middle, and end of the run were about 25.5°, 26.0°, and 25.0°C, respectively. After each 500 psi pressure change, about 15 min was allowed for the temperature to approach equilibrium before frequency readings were taken.

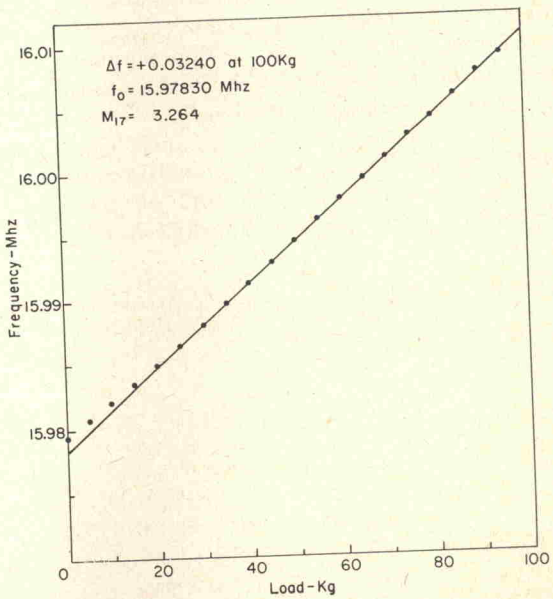

FIG. 2. Example of data for a uniaxial stress run. Some nonlinearity in the stress-frequency dependence at low stresses was often seen.

TABLE III. The measured slopes of the stress dependences of the second-order elastic constants for three independent sets of measurements and the third-order elastic constants determined from them. The third-order elastic constants listed as "best" values were obtained by combining all of the data, and the "best"-value slopes were then calculated from them. The relation numbers refer to the relations in order in Tables I–III of Ref. 26.

Relation No.	Sample 2	Sample 1 before irradiation	Sample 1 after irradiation	"Best" values
1	+7.59±0.13	a	(7.5621±0.13)	+7.5621
2	+0.221±0.008	a	(0.2235±0.006)	+0.2235
3	+6.90±0.12	+6.95±0.32	(6.9456±0.12)	+6.9456
4	+0.844±0.016	+0.837±0.004	(0.8400±0.004)	+0.8400
5	+0.232±0.008	+0.222±0.006	(0.2235±0.006)	+0.2235
6	a	a	a	+0.1138
7	a	a	a	+1.6082
8	a	a	a	−0.4432
9	small pos.	+0.85±0.4	not meas.	+0.8891
10	+0.856±0.013	+0.774±0.044	+0.767±0.016	+0.8328
11	−0.446±0.042	−0.475±0.050	−0.442±0.070	−0.4432
12	small neg.	a	a	+0.1138
13	−2.34±0.032	a	a	−2.3222
14	+3.83±0.09	a	a	+3.4871
15	small pos.	+1.2±0.7	not meas.	+1.4651
16	−0.49±0.06	−0.54±0.06	−0.502±0.032	−0.4929
17	+3.20±0.07	+3.14±0.05	+3.264±0.016	+3.2379
C_{111}	−25.630±0.33 [b]	−25.683±0.66	−25.736±0.26	−25.64±0.25
C_{112}	−11.387±0.27	−11.389±0.60	−11.355±0.24	−11.40±0.25
C_{123}	−4.622±0.33	−4.746±0.65	−4.769±0.26	−4.67±0.25
C_{144}	−3.444±0.11	−3.500±0.13	−3.384±0.17	−3.43±0.10
C_{166}	−1.677±0.06	−1.639±0.065	−1.701±0.09	−1.677±0.05
C_{456}	+1.360±0.16	+1.302±0.044	+1.387±0.031	+1.366±0.05
$C_{111}+2C_{112}$	−48.404±0.66	−48.460±1.68	not meas.	−48.45±0.66
$C_{144}+2C_{166}$	−6.799±0.040	−6.779±0.030	not meas.	−6.786±0.030
$C_{111}-C_{123}$	−21.008±0.16	−20.937±0.040	not meas.	−20.967±0.040

[a] These values could not be determined because of the crystallographic orientations of the samples.
[b] The units for all the third-order elastic constants are 10^{12} dyn/cm^2.

for the values of the three combinations of TOEC ($C_{111}+2C_{112}$), ($C_{144}+2C_{166}$), and ($C_{111}-C_{123}$), which were considered to be exact within the limits of their uncertainties based entirely on the uncertainties in the values of m_n estimated previously. Of the remaining uniaxial stress data, the data obtained using longitudinal ultrasonic waves were ignored because of their very large uncertainties, and data for relations (6), (7), and (8) could not be obtained because the samples were not of the right crystallographic orientation. The remaining relations, given in Table I, were then combined with the three relations obtained from the hydrostatic pressure data in such a manner as to obtain the best agreement with all the data taking into account their relative uncertainties.

The data for the polycrystalline samples were analyzed in a similar manner, again relying heavily on the hydrostatic pressure measurements.

TABLE IV. The adiabatic second-order elastic constants of two polycrystalline columbium samples at 298°K. A density of 8.578 g/cm^3 was used. The Voigt–Reuss–Hill average of the single-crystal elastic constants is shown for comparison.

		Sample A elongated 30×75 μ grains	Sample B equiaxed 10 μ grains	Calculated by VRH approx. from single xtal data
$C_{11}=\lambda+2\mu$	10^{12} dyn/cm^2	2.179±0.004	2.203±0.004	2.2113
$C_{12}=\lambda$	10^{12} dyn/cm^2	1.433±0.006	1.453±0.006	1.4597
$C_{44}=\mu$	10^{12} dyn/cm^2	0.373±0.001	0.375±0.001	0.3758
E	10^{12} dyn/cm^2	1.041	1.048	1.0505
G	10^{12} dyn/cm^2	0.373	0.375	0.3758
K	10^{12} dyn/cm^2	1.682	1.703	1.7102
σ—Poisson's ratio		0.397	0.397	0.3977